

Mid-Term Study Guide for Mh4714

The mid-term exam takes place at 5.00 p.m. on Tuesday 11th October in G37.

1. Questions such as those on Exercise sheets I,II,III and IV.
2. Definition of $\lim_{n \rightarrow \infty} a_n = L$.
3. Use the definition of limit to prove that $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$.
4. Use the definition of limit to prove that $\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1$.
5. Use the definition of limit to prove that the constant sequence $\{k\}$ has limit k .
6. Use the properties of limits and the above two limits to carefully find limits such as
$$\lim_{n \rightarrow \infty} \frac{3n^3 + n^2}{2n^3 + 3n} = \frac{3}{2}$$
7. Prove that a convergent sequence is bounded.
8. Prove the “Squeezing” theorem.
9. Given some alleged proofs of one of the above theorems identify which are correct and which aren't.
10. Prove that, if $k > 0$ then $(1+k)^n \geq 1+kn$, $\forall n \in \mathbb{N}$
11. Prove that a given sequence of the type $\{r^n\}$ with $|r| < 1$ converges to 0.
12. Prove that a given sequence of the type $\{r^n\}$ with $|r| > 1$ does not converge.
13. Show that a given infinite repeating decimal converges to a rational number.
14. Explain how we know that a series such as $\sum_{k=1}^{\infty} \frac{1}{10^{\frac{k}{k+1}}}$ does not converge to a rational number.